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The suppression of neutralino annihilation into Zh

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Abstract. The indirect detection of neutralino dark matter is most promising through annihilation channels producing a hard energy spectrum for the detected particles, such as neutralino annihilation into Zh. A cancellation however makes this particular annihilation channel generically subdominant in the huge parameter space of supersymmetric models. This cancellation requires non-trivial relations between neutralino mixings and masses, which we derive from gauge independence and unitarity of the MSSM. To show how the cancellation overshoots leaving only a subdominant result, we use a perturbative expansion in powers of the electroweak/supersymmetry breaking ratio m_Z/m_χ.

1 Introduction and motivations

There is no doubt that standard models of particle physics and cosmology alone cannot describe the full wealth of observational data recently collected on a wide variety of length scales. Ad hoc as it may seem, the dark matter (DM) hypothesis [1, 2] is probably part of the minimal set of extra ingredients needed to account for the increased gravitational self-attraction of matter on scales ranging from galaxies to the full visible universe. More exotic ingredients like repulsive dark energy, or modifications of gravity itself, might also become necessary to cope with the apparent acceleration of the universe. In the absence of a convincing unified theoretical solution to these both issues, experimental searches are the only way to prove the validity of hypotheses like the existence of a DM particle. For instance, the first issue would be settled if a new particle were found and its non-gravitational interactions measured to be compatible with the cold dark matter relic density required by the cosmic microwave background and large scale structure formation [3, 4]. However, these interactions are then by definition weak, and we should not be surprised that their evidence is extremely hard to obtain, much like for Pauli's neutrino. Debates like the one around DAMA's claim [5–12] for direct detection of DM are illustrative of this difficulty. This is why it is crucial to be able to cross-check and understand results in as many different ways as possible, for which a definite and well motivated DM theoretical framework is necessary. In this work, we shall keep with the well-studied supersymmetric lightest neutralino (χ) of mSugra or MSSM models.

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A particularly crucial cross-check would be the indirect detection of the neutralino annihilation products, which would show that such annihilation did indeed occur in the past, frozen out at some point and re-started in hot spots like the galactic center or the solar core, where dark matter later accumulated. However, to identify an indirect DM signal and possibly determine the neutralino mass by looking at fluxes of e.g. photons or neutrinos from such hot spots, it is essential to be able to distinguish that signal from the standard but poorly known astrophysical background. These being characterized by energy spectra with fairly universal power laws, indirect detection will be most successful when neutralino annihilation proceeds through primary channels which provide secondary photons or neutrinos with the hardest possible spectra, and a sharp energy cut-off around the neutralino mass. From Fig. 1 (discussed in Appendix A), the most promising annihilation channels are into a $\tau^+\tau^-$ pair, into two gauge bosons ($\chi\chi \to W^+W^-$, or ZZ, which has the same shape) or into one gauge boson and a Higgs–Englert–Brout (HEB) boson [13, 14] ($\chi\chi \to Zh$).

However, these fairly universal spectra need to be weighted by the actual model-dependent branching ratios to give the final indirect DM detection signal. It was noted long ago [15] that the Zh channel is then suppressed, which can be numerically checked[1] using the DarkSusy (3.14.02 version) [16] and the Suspect (2.003 version) code [17]: the top plot of Fig. 2 typically shows a suppression by three orders of magnitude for various mSugra models with $\tan\beta = 10$. To qualify this suppres-

[1] Temporarily failing such check after an update of the DarkSusy–Suspect interface was actually the starting point of this work.

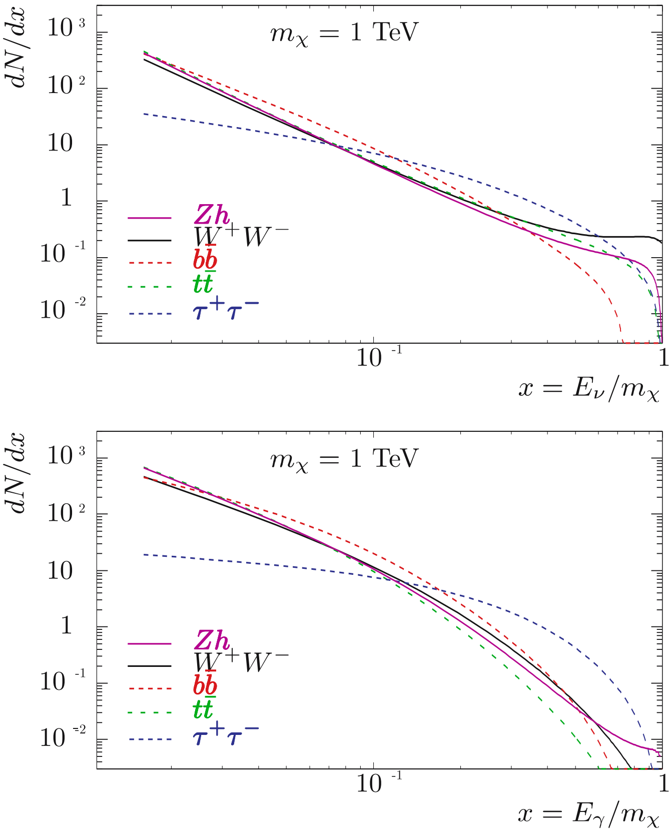


Fig. 1. The shape of differential neutrino (*top*) and gamma (*bottom*) fluxes coming from neutralino annihilations into various primary channels. The normalizations (depending on branching ratios) have been arbitrarily rescaled to compare the shape of all channels; the dependence on m_χ (1 TeV here) is weak

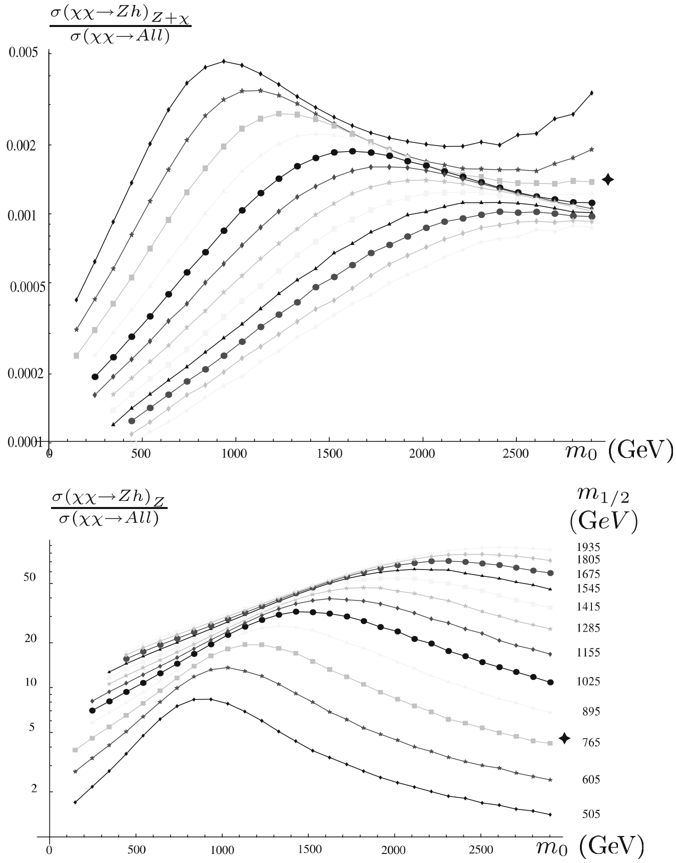


Fig. 2. Neutralino annihilation branching ratios to Zh for various m_0 and $m_{1/2}$ values: Z exchange only (*bottom*), Z and χ exchanges (*top*)

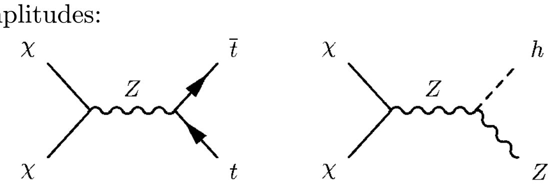
sion, let us start by contributions we expect to be dominant, namely those from SM particle exchanges (in our case an s-channel Z) since superpartners are necessarily heavier. As seen in the bottom panel of Fig. 2, this contribution not only dominates but even overwhelms the total cross-section. We therefore need to understand a double suppression, that first cancels this large contribution and second brings the total Zh annihilation below other channels. The cancelling contribution necessarily involves non-SM particle exchanges, which seems contradictory with the fact that on Fig. 2, the cancellation gets better with increasing m_0 and $m_{1/2}$, i.e. for maximally broken supersymmetry.

To be as general as possible, this cancellation should be checked in a supersymmetry breaking independent way. This is done in Fig. 3 in the more general MSSM, while keeping the usual GUT relation $M_1 = \frac{5}{3}\tan\theta_{\mathrm{W}} M_2 \simeq 0.5 M_2$. A cancellation up to three orders of magnitude thus seems a generic property of every broken supersymmetry theory.

To be more concrete, let us now focus on the particular mSugra model with $m_0 = 3000$ GeV, $m_{1/2} = 800$ GeV, $A_0 = 0$, $\tan(\beta) = 10$ and $\mu > 0$, marked by the black star in Fig. 2. The annihilation cross-sections at rest are:

$v\sigma$ (cm^3/s)	$\chi\chi \to t\bar{t}$	$\chi\chi \to Zh$
Z exchange	1.83×10^{-28}	4.72×10^{-28}
All diagrams	1.03×10^{-28}	1.48×10^{-31}
	$v\sigma\,(\chi\chi \to \text{all}) = 1.05 \times 10^{-28}$	

The annihilation is dominated by the $t\bar{t}$ channel, three orders of magnitude larger than the Zh one [18]. However, when restricting to the Z exchange diagram, they are comparable, with Zh slightly larger. This is easily understood by exhibiting the couplings and kinematic factors in the amplitudes:



$$A\left(\chi\chi \to t\bar{t}\right)_Z \propto \frac{-g^2 m_t O^Z_{11}}{\cos^2\theta_{\mathrm{W}}}\,,\quad A\left(\chi\chi \to Zh\right)_Z \propto \frac{-g^2 m_Z O^Z_{11}}{\cos^2\theta_{\mathrm{W}}}\,,$$

where the $\chi\chi Z$ coupling O^Z_{11} is defined in terms of the neutralino mixing matrix[2] N (see Appendix D.1):

[2] In what follows, we will always work with the hypothesis of absence of CP-violating phases, in order for the N matrix to be real (see Appendix C).

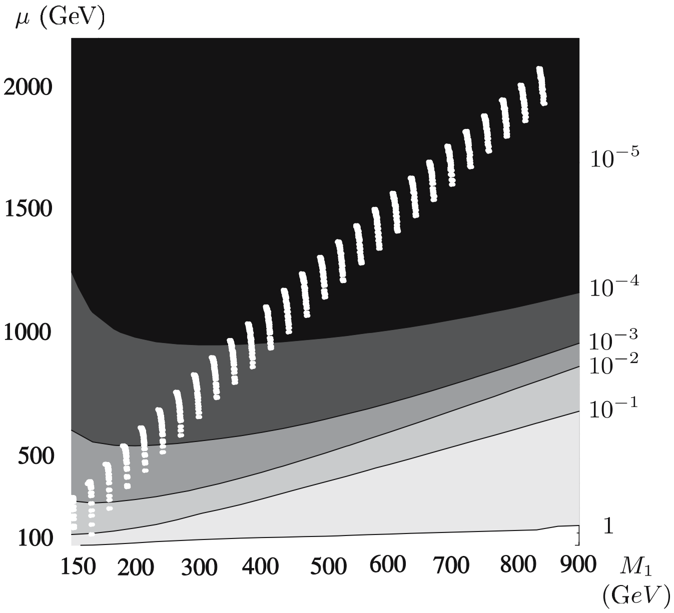


Fig. 3. Contour plot of $\frac{\sigma(\chi\chi\to Zh)_{Z+\chi}}{\sigma(\chi\chi\to Zh)_Z}$ in the MSSM; *white dots* correspond to the mSugra models of Fig. 2

$$O^Z_{ij} = (N_{i4}N_{j4} - N_{i3}N_{j3}) \ . \tag{1}$$

It remains to understand why other exchanges cancel the Zh channel and not the $t\bar{t}$ one. For the latter, the t-channel sfermion exchange can be made arbitrarily small by taking large enough m_0, so that the main contribution proceeds via (SM) Z boson exchange, conforming to naive expectations. However, two other diagram are involved in the Zh annihilation channel: (1) an s-channel pseudoscalar A exchange, which can be neglected for large m_0 (in what follows we will always work in this pseudoscalar decoupling limit (D.5)), and (2) t-channel exchanges of the four neutralinos, which cannot be decoupled because of strong links between the couplings involved in each diagram. These links appear in the expression of the annihilation amplitudes [19, 20] derived in the next section:

$$A\,(\chi\chi \to Zh)_Z = \frac{-\mathrm{i}g^2\sqrt{2}}{\cos^2\theta_\mathrm{W}}\frac{m^2_\chi}{m^2_Z}\beta_{Zh} \times O^Z_{11} \tag{2}$$

$$A\,(\chi\chi \to Zh)_\chi = \frac{\mathrm{i}g^2\sqrt{2}}{\cos^2\theta_\mathrm{W}}\frac{m^2_\chi}{m^2_Z}\beta_{Zh} \times \sum_{i=1}^{4}\frac{2O^Z_{1i}O^h_{1i}\left(m_{\chi_i} - m_\chi\right)m_Z}{2m^2_\chi + 2m^2_{\chi_i} - m^2_h - m^2_Z}\ , \tag{3}$$

where the $\chi_i\chi_j h$ coupling O^h_{ij} is defined in (D.7) as

$$O^h_{ij} = (c_\mathrm{W}N_{i2} - s_\mathrm{W}N_{i1})\,(s_\beta N_{j4} - c_\beta N_{j3}) + (i \leftrightarrow j)\ , \tag{4}$$

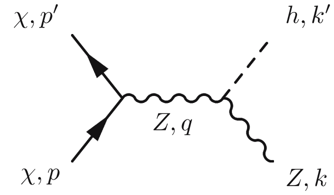
where the decoupling condition (D.5) has been used. It is clear that the second amplitude, (3), cannot easily be neglected and might turn out to be comparable with the first one, (2). However, it is less clear why both should cancel with high precision, especially given as different-looking couplings as (1) and (4).

A toy example of the above cancellation is provided by the annihilation of a spin singlet $t\bar{t}$ pair into Zh in a standard model without $SU(2)$. One may first be puzzled to get a cancellation between an s-channel Z exchange, which only involves gauge couplings, and a t-channel top exchange, which involves an a priori independent Yukawa coupling. However, one soon realizes that the existence of the s-channel requires both spontaneous breaking of the gauge symmetry for the ZZh vertex and an axial coupling for the $t\bar{t}Z$ vertex. This last coupling excludes contributions to the top mass other than the gauge breaking $y\langle h\rangle$ one. The two channels $g\frac{1}{m^2_Z}gm_z \propto g\frac{1}{m_t}y$ are then both proportional to $g/\langle h\rangle$ and may cancel. In contrast with this simple case, the neutralino annihilation studied below is complicated by the presence of another source of mass, namely the SUSY breaking Majorana mass terms for the gauginos.

To analyze this cancellation, we start in Sect. 2 by deriving the relevant amplitudes. In Sect. 3, gauge independence is used to draw a first link between the couplings, which is shown to follow from the gauge invariance of the mass matrix. Unitarity at high energy is then used in Sect. 4 to derive a second relation, which is combined with the first one to show that a cancellation is possible. In Sect. 5, the structure of Rayleigh–Schroedinger perturbation theory is then used to show that this cancellation is stronger than expected.

2 Amplitudes for neutralino annihilation at rest

Let us start by deriving the polarized amplitude $A\,(Zh)_Z$ for neutralino annihilation into Zh via Z exchange. By construction, neutralinos are their own antiparticles, so that an initial pair of lightest neutralinos at rest is necessarily in an antisymmetric spin singlet state. The final state containing a HEB scalar, the outgoing Z boson polarization needs to be longitudinal and can be chosen as z-axis. It is then more convenient to use helicity amplitudes [15, 19] than unpolarized cross-sections [20]. The Feynman diagram and rules defined in Appendix D give the amplitude



$$A\,(\chi\overline{\chi} \to Zh)_Z = \frac{-\mathrm{i}C^A_{11}C_Z}{q^2 - m^2_Z}\left(\overline{\chi}\,(p')\,\gamma^\mu\gamma_5\chi\,(p)\right) \times \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m^2_Z}\right)\varepsilon^\nu\,(k)\ , \tag{5}$$

where $\chi = u$ is the incoming neutralino, $\overline{\chi} = \overline{v}$ the outgoing one in an arbitrary choice of arrows directions. u and

v are external Dirac spinors at rest in the chiral basis (see Appendix B), namely

$$\chi\left(p_0\right)=\sqrt{m_\chi}\begin{pmatrix}\xi_s\\ \xi_s\end{pmatrix};\ \text{with}\ \xi_{+\frac{1}{2}}=\begin{pmatrix}-1\\ 0\end{pmatrix}\ \text{or}\ \xi_{-\frac{1}{2}}=\begin{pmatrix}0\\ 1\end{pmatrix}$$

for a spin up or down along the z-axis, and similarly for the "antiparticle":

$$\overline{\chi}\left(p_0'\right)=\sqrt{m_\chi}\left(-\eta_{s'},\eta_{s'}\right).$$

Thanks to the Majorana condition (B.4), descriptions of the same neutralino as a particle with $\xi_{s'}$ or as an "antiparticle" with $\eta_{s'}$ are equivalent, provided

$$\eta_{s'}=-\mathrm{i}\sigma^2\xi_{s'}\,. \tag{6}$$

The polarization of the longitudinal Z boson being $\varepsilon^\nu(k)=(k^z,0,0,k^0)/m_Z$, and the initial momentum at rest simply $q^\mu=(2m_\chi,0)$, the polarized amplitude (5) is

$$A\left(\chi_\uparrow\chi_\downarrow\to Zh\right)_Z=\frac{\mathrm{i}C_{11}^A C_Z}{4m_\chi^2-m_Z^2}\frac{m_\chi}{m_Z}\left(\sigma_{11}^\mu+\bar{\sigma}_{11}^\mu\right) \tag{7}$$

$$\times\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_Z^2}\right)\left(k^z,0,0,k^0\right)$$

$$=-2\mathrm{i}C_{11}^A C_Z\frac{m_\chi}{m_Z^3}k^z\,. \tag{8}$$

Notice the time-like structure of the initial state vector $(\sigma_{11}^\mu+\bar{\sigma}_{11}^\mu)=(2,0,0,0)$: a purely axial coupling talks only with the scalar part of the two spins at rest. Notice also the disappearing of the Z pole at $m_\chi=m_Z/2$.

Expressing $k^z=m_\chi\beta_{Zh}$ in terms of the conventional kinematic factor

$$\beta_{Zh}=\sqrt{1-\frac{\left(m_h+m_Z\right)^2}{4m_\chi^2}}\sqrt{1-\frac{\left(m_h-m_Z\right)^2}{4m_\chi^2}}\underset{m_\chi\gg m_Z}{\approx}1$$

and using the definitions (D.1) and (D.4) of the couplings in terms of neutralino mixings given in Appendix D, we finally find

$$A\left(\chi_\uparrow\chi_\downarrow\to Zh\right)_Z=\frac{-\mathrm{i}g^2 O_{11}^Z}{\cos^2\theta_{\mathrm{W}}}\frac{m_\chi^2}{m_Z^2}\beta_{Zh}\,. \tag{9}$$

To get the amplitude with reversed helicities, we just need to replace the $(1,1)$ components of the Pauli matrices in (7) by the $(2,2)$ components and take the extra sign from the Majorana condition (6) into account. Thanks to this sign, only an antisymmetric initial state can contribute, and it gives with the correct normalization factor

$$A\left(\chi\chi\to Zh\right)_Z=\frac{-\mathrm{i}g^2\sqrt{2}}{\cos^2\theta_{\mathrm{W}}}\frac{m_\chi^2}{m_Z^2}\beta_{Zh}O_{11}^Z\,, \tag{10}$$

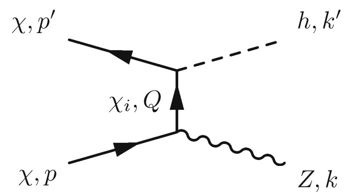
in agreement with (2) and existing results [19, 20].

This amplitude (10) has to be compared with the result of a similar computation for the annihilation into $t\bar{t}$:

$$A\left(\chi\chi\to t\bar{t}\right)_Z=\frac{-\mathrm{i}g^2\sqrt{2}}{\cos^2\theta_{\mathrm{W}}}\frac{m_\chi m_t}{m_Z^2}\beta_{t\bar{t}}T_3 O_{11}^Z\,, \tag{11}$$

where $\beta_{t\bar{t}}=\sqrt{1-m_t^2/m_\chi^2}$ and $T_3=1$ is the weak isospin of the top quark. Notice the different power of m_χ, favoring the Zh channel for large masses.

We have seen in the introduction that t-channel neutralinos exchanges should reverse this conclusion. Following the same path, their contribution is seen to be



$$A\left(\chi\chi\to Zh\right)_{\chi_i}=\frac{-\mathrm{i}C_{1i}^A C_{1i}^h}{Q^2-m_{\chi_i}^2}$$

$$\times\left(\overline{\chi}\left(p'\right)\left(\not{Q}+m_{\chi_i}\right)\gamma^\mu\gamma_5\chi\left(p\right)\right)\varepsilon_\mu\left(k\right),$$

with $Q^\mu=p^\mu-k^\mu$. After some algebra on the t-channel, and adding the u-contribution (same diagram as before with p and p' interchanged), one finds

$$A\left(\chi\chi\to Zh\right)_{\chi_i}=\frac{\mathrm{i}g^2\sqrt{2}}{\cos^2\theta_{\mathrm{W}}}\frac{m_\chi^2}{m_Z^2}\beta_{Zh} \tag{12}$$

$$\times\frac{2O_{1i}^Z O_{1i}^h\left(m_{\chi_i}-m_\chi\right)m_Z}{2m_\chi^2+2m_{\chi_i}^2-m_Z^2-m_h^2}\,.$$

The problem is now to understand how the amplitudes (12) and (10) cancel with the 10^{-3} precision shown in the introduction. This can happen only if the sum of the second lines of (12) which contain four powers of the neutralino mixing matrix N somehow reduces to $O_{11}^Z\sim N^2$ as a consequence of some symmetry. We already noticed that supersymmetry had to be maximally broken for the cancellation to take place. We are thus left with gauge invariance which is investigated in the next section.

3 Gauge independence and gauge invariance of the mass matrix

The previous computations were performed in the unitary gauge, where massive gauge fields have completely "eaten" a Goldstone boson. One way to obtain non-trivial relations of the kind we seek is to work in the R_ξ-gauge family of 't Hooft [21], and require independence of the result on the gauge-fixing parameter ξ.

Let us first notice that the neutralino exchange diagrams are gauge independent by themselves. Indeed, in a vector supermultiplet, only the bosonic gauge field is gauge dependent, and not the associated gaugino. Moreover, higgsinos are associated with the real part of complex scalars, which are also gauge independent.

We can thus concentrate on the Z exchange diagram. When going from unitary to R_ξ-gauges, this contribution splits in two ξ-dependent diagrams: one with Goldstone boson exchange, and another with the Z boson exchange.

To exhibit the cancellation of their ξ dependence, it is convenient in the Z propagator

$$\frac{-\mathrm{i}}{q^2-m_Z^2}\left(g_{\mu\nu}-\frac{(1-\xi)\,q_\mu q_\nu}{q^2-\xi m_Z^2}\right)$$

to decompose the longitudinal part as

$$\frac{m_Z^2\,(1-\xi)}{\left(q^2-m_Z^2\right)\left(q^2-\xi m_Z^2\right)}=\frac{1}{\left(q^2-m_Z^2\right)}-\frac{1}{\left(q^2-\xi m_Z^2\right)}\,. \tag{13}$$

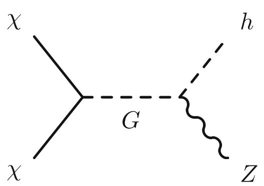
The first term is nothing but the longitudinal unitary gauge propagator, and the second exhibits a fake pole at the Goldstone mass $q^2=\xi m_Z^2$, with a wrong sign [22]. The ξ-dependent Z exchange $A_{Z\xi}=A_Z+A_{ZG}$ correspondingly decomposes into the previously obtained A_Z (5) and a Goldstone-like amplitude with gauge couplings:

$$A\,(\chi\chi\to Zh)_{ZG}=-\mathrm{i}\frac{C^A C_Z}{m_Z^2}\left(\overline{\chi}\,(p')\,\not{q}\gamma_5\chi\,(p)\right)\frac{q\cdot\varepsilon\,(k)}{q^2-\xi m_Z^2}$$

which, using the mass-shell condition for the initial state $\bar{\chi}\not{q}\gamma_5\chi=2m_\chi\bar{\chi}\gamma_5\chi$, becomes

$$A\,(\chi\chi\to Zh)_{ZG}=\frac{-\mathrm{i}g^2 O_{11}^Z}{2\cos^2\theta_{\mathrm{W}}}\frac{m_\chi}{m_Z}\overline{\chi}\gamma_5\chi\frac{q\cdot\varepsilon\,(k)}{q^2-\xi m_Z^2}\,. \tag{14}$$

We now turn to the genuine Goldstone boson exchange. The ZZh coupling is replaced by the GZh one:



$$C_{Gh}^Z=\mathrm{i}C_G\,(k-k')_\mu\,, \tag{15}$$

where $C_G=g/2\cos\theta_{\mathrm{W}}$, and the scalar propagator is simply $\mathrm{i}/(q^2-\xi m_Z^2)$. But things are more subtle for the $G\chi\chi$ vertex: even if the neutral Goldstone boson is part of the Z boson in the unitary gauge, it does not have the same coupling to neutralinos:

$$C_{\chi_i\chi_j}^G=\frac{\mathrm{i}gO_{ij}^G}{2\cos\theta_{\mathrm{W}}}\,, \tag{16}$$

with

$$O_{ij}^G=(N_{i2}c_{\mathrm{W}}-N_{i1}s_{\mathrm{W}})\,(c_\beta N_{j3}+s_\beta N_{j4})+(i\leftrightarrow j)\,. \tag{17}$$

For the lightest neutralino annihilation, $i=j=1$, and since the coupling is imaginary, the Goldstone only couples to the axial part of the neutralino. The amplitude for the Goldstone exchange diagram is thus

$$\begin{aligned}A\,(\chi\chi\to Zh)_G=&-C_{\chi\chi}^G C_G\,\overline{\chi}\,(p')\,\gamma_5\chi\,(p)\\&\times\frac{(q+k')_\mu}{q^2-\xi m_Z^2}\varepsilon^\mu\,(k)\,.\end{aligned} \tag{18}$$

Recalling the kinematics ($q=k+k'$), the polarization condition ($k\cdot\varepsilon\,(k)=0$), and the definitions of the couplings, we finally have

$$A\,(\chi\chi\to Zh)_G=\mathrm{i}\frac{g^2O_{11}^G}{2c_{\mathrm{W}}^2}\overline{\chi}\gamma_5\chi\,\frac{q\cdot\varepsilon\,(k)}{q^2-\xi m_Z^2}\,. \tag{19}$$

Now, comparing (14) and (19), gauge independence requires a relation between the couplings:

$$O_{11}^Z\frac{m_\chi}{m_Z}=-\frac{1}{2}O_{11}^G\,,$$

which by (1) and (17) can be expressed in terms of the neutralino mixings and masses:

$$\begin{aligned}&\left(N_{14}^2-N_{13}^2\right)\frac{m_\chi}{m_Z}\\&\quad=-\left(N_{12}c_{\mathrm{W}}-N_{11}s_{\mathrm{W}}\right)\times\left(s_\beta N_{14}+c_\beta N_{13}\right)\,.\end{aligned} \tag{20}$$

This relation can be extended for the annihilation of an arbitrary pair of neutralinos χ_i–χ_j in the same channel:

$$O_{ij}^Z\frac{m_{\chi_i}+m_{\chi_j}}{m_Z}=-O_{ij}^G\,,$$

which is a short version of the rather non-trivial identity:

$$\begin{aligned}&(N_{i4}N_{j4}-N_{i3}N_{j3})\,\frac{m_{\chi_i}+m_{\chi_j}}{m_Z}\\&\quad=-\left(N_{i2}c_{\mathrm{W}}-N_{i1}s_{\mathrm{W}}\right)(s_\beta N_{j4}+c_\beta N_{j3})-(i\leftrightarrow j)\,.\end{aligned} \tag{21}$$

Although not completely identical, this relation bears similarities with the combination of masses and mixings involved in (3). It is therefore interesting to notice that it only involves the neutralino mass matrix and can be derived in the following way. By the definition of the mixing matrix N, (C.2), we have

$$(NM)_{ij}=m_iN_{ij}\,.$$

Then for any matrix P, the following identities hold:

$$(m_i+m_j)\left(NPN^{-1}\right)_{ij}=\left(N\,(MP+PM)\,N^{-1}\right)_{ij}\,. \tag{22}$$

As a particular case, if we take for P the isospin operator that flips the first higgsino sign compared to the second one,

$$P=T_3=\begin{pmatrix}0&0\\0&-\sigma_3\end{pmatrix}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}\,,$$

we recover the gauge independence relation (21). The particular form of the right-hand side of this relation then follows from the special structure of the symmetric neutralino mass matrix:

$$M = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix} ,$$

where A is a 2×2 diagonal matrix reflecting the Majorana nature of gauginos, B is a 2×2 anti-diagonal matrix reflecting the Dirac nature of the $SU(2)$-charged higgsino pair, and $C \sim m_Z$ is a 2×2 matrix that vanishes when $SU(2)$ is unbroken and with null determinant to otherwise ensure masslessness of the photon. These conditions make $PM + MP$ off-diagonal, explaining the non-trivial vanishing of the left-hand side of (22) with m_Z.

These remarks stress the central role played by gauge invariance in the structure of the mass matrix. The appearance of $P = T_3$ is no surprise, as all $\chi\chi Z$ couplings find their root in the gauge invariant $\tilde{h}\tilde{h}Z$: higgsinos can couple to the Z boson thanks to their $U_Y(1)$ charge which dictates a Dirac behavior. Only the spontaneous breaking of $SU(2)$ and $U_Y(1)$ carried by C can then split this degenerate Dirac system into a pair of Majorana particles.

4 High energy unitarity

Despite vague similarities, the gauge independence relations (21) do not yet explain the cancellation of (2) and (3), and in particular their different powers of mixings N_{ij}. To get further, we need another relation. Using the pinch technique [23], we therefore turn to the high energy behavior of the amplitude. Indeed, from rotation invariance, the outgoing Z boson must have a pure longitudinal polarization, and it is well known that this can lead to conflicts with the perturbative unitarity constraint that scattering amplitudes be bounded by a constant, $A(s \to \infty) < K$.

Having checked gauge independence, we can for simplicity use the Feynman gauge $\xi = 1$ to have a purely transverse Z propagator free from high energy divergences. The dangerous diagrams which must cancel are then the s-channel Goldstone and t-channel neutralinos exchanges.

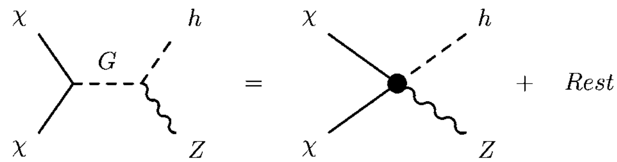
A light-like vector being orthogonal to itself, the polarization vector at high energies is approximately

$$\varepsilon_\mu(k) \simeq \frac{k_\mu}{m_Z}. \tag{23}$$

Using this and the kinematic identity $2q\cdot k = q^2 + m_Z^2 - m_h^2$, the amplitude for Goldstone exchange (19) in this gauge becomes

$$A\left(\chi\chi \to Zh\right)_G = \frac{\mathrm{i}g^2}{2\cos^2\theta_{\mathrm{W}}} \frac{O_{11}^G}{2m_Z} \overline{\chi}\gamma_5\chi \times \left(1 + \frac{2m_Z^2 - m_h^2}{q^2 - m_Z^2}\right) . \tag{24}$$

The first "contact" term in the parentheses gives a contribution $A \simeq \sqrt{s} = \sqrt{q^2}$, which is divergent and violates unitarity in the high energy limit, while the second term is better behaved thanks to the appearance of a propagator denominator. From a diagrammatic point of view, this is expressed by splitting the diagram into a "pinched" part and a rest:



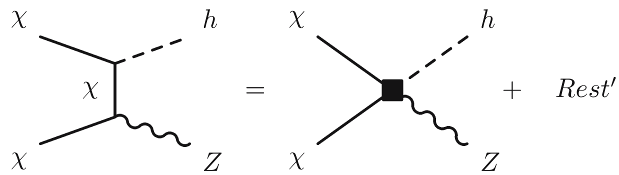
For the neutralino exchange channel, we have

$$A\left(\chi\chi \to Zh\right)_\chi = -2\mathrm{i}\sum_{i=1}^{4} \frac{C_{1i}^A C_{1i}^h}{Q^2 - m_{\chi_i}^2} \times \overline{\chi}\left(p'\right)\left(\not{Q} + m_{\chi_i}\right)\gamma^\mu\gamma_5\chi\left(p\right)\frac{k_\mu}{m_Z} .$$

Expressing $\not{k} = \not{p} - \not{Q} = (\not{p} - m_\chi) - (\not{Q} - m_{\chi_i}) + (m_\chi - m_{\chi_i})$, the first term vanishes on-shell, while the second cancels the propagator pole to give a contact term and a rest:

$$A\left(Zh\right)_\chi = \frac{-\mathrm{i}g^2}{2c_{\mathrm{W}}^2}\sum_{i=1}^{4}\frac{O_{1i}^Z O_{1i}^h}{m_Z} \times \left\{\overline{\chi}\gamma_5\chi - \frac{m_\chi - m_{\chi_i}}{Q^2 - m_{\chi_i}^2}\,\overline{\chi}\left(\not{Q} + m_{\chi_i}\right)\gamma_5\chi\right\} . \tag{25}$$

which can be diagrammatically represented by



Cancellation of the contact terms in (24) and (25) requires the following identity to hold:

$$\frac{1}{2}O_{11}^G = \sum_i O_{1i}^Z O_{1i}^h ,$$

or, using (1), (4) and (17), the following relation among the mixings N:

$$\begin{aligned} &\left(N_{12}c_{\mathrm{W}} - N_{11}s_{\mathrm{W}}\right)\left(s_\beta N_{14} + c_\beta N_{13}\right) \\ &= \sum_{i=1}^{4}\left(N_{14}N_{i4} - N_{13}N_{i3}\right) \\ &\quad \times \left\{\left(c_{\mathrm{W}}N_{12} - s_{\mathrm{W}}N_{11}\right)\left(N_{i4}s_\beta - N_{i3}c_\beta\right)\right. \\ &\quad \left. + \left(c_{\mathrm{W}}N_{i2} - s_{\mathrm{W}}N_{i1}\right)\left(N_{14}s_\beta - N_{13}c_\beta\right)\right\} . \end{aligned} \tag{26}$$

As complicated as it may seem, this equation simply follows from the orthogonality condition $N_{ij}N_{kj} = \delta_{ik}$, with $i, k = 3, 4$. We have therefore shown that high energy perturbative unitarity of the amplitude is guaranteed by the unitarity of the mixing matrix.

By combining (26) and (20), a new non-trivial identity among couplings is obtained:

$$\frac{m_\chi}{m_Z}O_{11}^Z = -\sum_i O_{1i}^Z O_{1i}^h ,$$

which is even less trivial when using the definitions of O^Z_{ij} and O^h_{ij}:

$$\frac{m_\chi}{m_Z}\left(N_{i4}^2-N_{i3}^2\right)=-\sum_{i=1}^{4}\left(N_{14}N_{i4}-N_{13}N_{i3}\right) \times\left\{\left(c_\mathrm{W}N_{12}-s_\mathrm{W}N_{11}\right)\left(N_{i4}s_\beta-N_{i3}c_\beta\right)+\left(c_\mathrm{W}N_{i2}-s_\mathrm{W}N_{i1}\right)\left(N_{14}s_\beta-N_{13}c_\beta\right)\right\}. \tag{27}$$

This identity relates the couplings appearing in Z exchange (2) and χ exchange (3) and suggests us to rewrite the second as

$$A\left(\chi\chi\to Zh\right)_\chi=-2\mathrm{i}\sqrt{2}\beta_{Zh}\frac{m_\chi}{m_Z}\frac{g^2}{\cos^2\theta_\mathrm{W}} \times\left(\frac{m_\chi}{m_Z}O^Z_{11}+\sum_{i=1}^{4}R_i\right), \tag{28}$$

where the first term exactly cancels the s-channel Z exchange, and the second is subdominant in the high energy limit $s\gg m_Z^2, m_\chi^2$. It is however not clear why those remainders should be subdominant at rest, because even for large $s=4m_\chi^2$, there is confusion between dynamically suppressed contributions $\sim m_Z^2/s\ll 1$ and neutralino mixing suppressed ones $\sim m_Z^2/m_\chi^2\ll 1$. Moreover, the definition

$$R_i=O^Z_{1i}O^h_{1i}\frac{\left(2m_{\chi i}m_\chi-4m_\chi^2-2m_{\chi i}^2+m_h^2+m_Z^2\right)}{2m_\chi^2+2m_{\chi i}^2-m_h^2-m_Z^2} \tag{29}$$

implies that for $m_Z\ll m_\chi$, $R_3\approx -R_4$ are comparable with the first term in (28), and only their sum becomes negligible. To analyze this final cancellation, a perturbative expansion in m_Z/m_χ is therefore needed.

5 Perturbation theory

In the SUSY decoupling limit ($m_Z\ll M_1, M_2,\mu$), the neutralino mass in Appendix C is naturally split [24] into $M=M_0+W$, with a leading contribution

$$M_0=\begin{pmatrix}M_1&0&0&0\\0&M_2&0&0\\0&0&0&-\mu\\0&0&-\mu&0\end{pmatrix},$$

and a perturbation

$$W=m_Z\begin{pmatrix}0&0&-s_\mathrm{W}c_\beta&s_\mathrm{W}s_\beta\\0&0&c_\mathrm{W}c_\beta&-c_\mathrm{W}s_\beta\\-s_\mathrm{W}c_\beta&c_\mathrm{W}c_\beta&0&0\\s_\mathrm{W}s_\beta&-c_\mathrm{W}s_\beta&0&0\end{pmatrix},$$

triggered by EW symmetry breaking. Following a standard Rayleigh–Schroedinger perturbation expansion, we start by solving the unperturbed eigensystem

$$N^0M_0N^{0\,\mathrm{T}}=m^0,$$

whose eigenvalues are $m^0=\mathrm{diag}\left(M_1,M_2,-\mu,\mu\right)$, and whose eigenvectors φ^0_n form the mixing matrix

$$N^{0\,\mathrm{T}}=\left(\varphi^0_1,\varphi^0_2,\varphi^0_3,\varphi^0_4\right)=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&\frac{1}{\sqrt{2}}&\frac{-1}{\sqrt{2}}\\0&0&\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}\end{pmatrix}.$$

The first order corrections to the eigenvalues $m^1_n=\left\langle\varphi^0_n\right|W\left|\varphi^0_n\right\rangle$ are simply the diagonal elements of $W^0=N^0W\,N^{0\,\mathrm{T}}$:

$$W^0=m_Z\begin{pmatrix}0&0&\frac{s_\mathrm{W}s_-}{\sqrt{2}}&\frac{s_\mathrm{W}s_+}{\sqrt{2}}\\0&0&-\frac{c_\mathrm{W}s_-}{\sqrt{2}}&-\frac{c_\mathrm{W}s_+}{\sqrt{2}}\\\frac{s_\mathrm{W}s_-}{\sqrt{2}}&-\frac{c_\mathrm{W}s_-}{\sqrt{2}}&0&0\\\frac{s_\mathrm{W}s_+}{\sqrt{2}}&-\frac{c_\mathrm{W}s_+}{\sqrt{2}}&0&0\end{pmatrix}, \tag{30}$$

where $s_\pm=s_\beta\pm c_\beta$. From the structure of the perturbation W, these diagonal elements clearly vanish. Furthermore, first order corrections to the eigenvectors

$$\left|\varphi^1_i\right\rangle=\sum_{j\neq i}\frac{W^0_{ji}}{m^0_i-m^0_j}\left|\varphi^0_j\right\rangle$$

can be regrouped into $N^{1\,\mathrm{T}}=\left(\varphi^1_1,\varphi^1_2,\varphi^1_3,\varphi^1_4\right)$ with

$$N^1=m_Z\times\begin{pmatrix}0&0&-\frac{s_\mathrm{W}C_1}{M_1^2-\mu^2}&\frac{s_\mathrm{W}S_1}{M_1^2-\mu^2}\\0&0&\frac{c_\mathrm{W}C_2}{M_2^2-\mu^2}&-\frac{c_\mathrm{W}S_2}{M_2^2-\mu^2}\\-\frac{s_\mathrm{W}s_-}{\sqrt{2}(M_1+\mu)}&\frac{c_\mathrm{W}s_-}{\sqrt{2}(M_2+\mu)}&0&0\\\frac{s_\mathrm{W}s_+}{\sqrt{2}(\mu-M_1)}&\frac{c_\mathrm{W}s_+}{\sqrt{2}(M_2-\mu)}&0&0\end{pmatrix}$$

and $C_{1,2}=\left(\mu s_\beta+M_{1,2}c_\beta\right)$, $S_{1,2}=\left(M_{1,2}s_\beta+\mu c_\beta\right)$.

For a bino-like neutralino ($M_1<M_2,\mu$), the $Z\chi\chi$ vertex does not exist without electroweak symmetry breaking, and knowing the diagonalization matrix $N=N^0+N^1$ up to first order in m_Z allows one in fact to compute the first non-trivial contribution to Z exchange (2) which appears at second order:

$$A\left(\chi\chi\to Zh\right)_Z=-\sqrt{2}\beta_{Zh}\frac{M_1^2}{m_Z^2}g^2c_{2\beta}t_W^2\frac{m_Z^2}{\left(M_1^2-\mu^2\right)}. \tag{31}$$

At the same order, the first non-trivial contribution to χ exchange (3) requires only one perturbation of the $Z\chi\chi_i$ vertex, and no perturbation of the $h\chi\chi_i$ vertex, which does exist in the absence of electroweak symmetry breaking. Further expanding the propagator in powers of m_Z would give the same structure as (28):

$$A\left(\chi \to Zh\right)_\chi = \sqrt{2}\beta_{Zh}\frac{M_1^2}{m_Z^2}g^2\cos 2\beta\tan^2\theta_{\mathrm{W}} \times \frac{m_Z^2}{(M_1^2-\mu^2)}\left(1+\frac{1}{2}\frac{m_Z^2+m_h^2}{M_1^2+\mu^2}\right), \tag{32}$$

suggesting that the remainder is $O(m_Z^4)$, i.e. two orders lower than the leading term. However, to firmly establish this conclusion requires a justification for the absence of terms $O(m_Z^3)$, which we shall now find by examining the general structure of the perturbative expansion.

When solving the eigensystem

$$\left(M_o+W\right)_{ij}N_{jl}^{\mathrm{T}} = N_{il}^{\mathrm{T}}m_l \tag{33}$$

by power expansions in m_Z for eigenvalues and eigenvectors,

$$m_i = m_i^0+m_i^1+\dots \tag{34}$$

$$N = N^0+N^1+\dots, \tag{35}$$

we can to all orders choose the correction to an eigenvector in $N-N^0$ orthogonal to the corresponding unperturbed vector in N^0: the only price is that we end up with non-unit vectors in N. This choice however simplifies the recurrence relation for the solution at order q to

$$m_i^q = \left(N^0WN^{q-1\,\mathrm{T}}\right)_{ii}, \tag{36}$$

$$\left(N^0N^{q\,\mathrm{T}}\right)_{ji} = \frac{\left(N^0WN^{q-1\,\mathrm{T}}\right)_{ji}}{m_i^0-m_j^0} - \sum_{p=1}^{q-1} m_i^p\frac{\left(N^0N^{q-p\,\mathrm{T}}\right)_{ji}}{m_i^0-m_j^0}.$$

The expressions for $q=0$ and $q=1$ were given above. We saw that M_0 is 2×2 block diagonal, and so is N^0, whereas W and thus N^1 are block off-diagonal. Following the recurrence, this can be generalized to show that N^q must be block diagonal for q even and block off-diagonal for q odd. Because of this structure, m_i^q will vanish for q odd, so that $m_i(m_Z)$ is holomorphic in m_Z^2. In a similar way, diagonal blocks of N have a purely even power expansion in m_Z, whereas off-diagonal ones only contain odd powers.

These results can be extended to the amplitudes A_Z and A_χ which then contain only even powers of m_Z: the lowest order $O(m_Z^{-2})$ vanishes for both as it should to allow for a smooth $m_Z\to 0$ limit; the next $O(m_Z^0)$ is equal and opposite for s- and t-channel exchanges, and the remainder $O(m_Z^2)$ dictates the amplitude of the Zh annihilation channel at rest to be lower than the $t\bar{t}$ pair channel. This can be loosely expressed as

$$A\left(Zh\right) \propto \frac{m_Z^2}{m_\chi^2} \ll A\left(t\bar{t}\right) \propto \frac{m_t}{m_\chi} \ll A\left(Zh\right)_Z \propto 1\,.$$

The order of magnitude and the power of the suppression,

$$\frac{\sigma\left(\chi\chi\to Zh\right)_{Z+\chi}}{\sigma\left(\chi\chi\to Zh\right)_Z} \propto \frac{m_Z^4}{M_1^4},$$

then agree with those displayed in Figs. 2 and 3.

6 Conclusion

In this work, we have given a quantitative understanding of why neutralinos at rest cannot annihilate predominantly into Zh. The possible relevance of this process for indirect DM detection has been shown in the introduction, before pointing out similarities and differences between the $t\bar{t}$ and Zh annihilation channels. We also stressed how much a naive estimate of this last channel can fail by ignoring the subtle but tight links between couplings imposed by symmetries, especially broken ones. Because of these links, a fine cancellation does occur which requires a closer look. Having noticed that this cancellation was getting finer with increasingly broken supersymmetry, we showed that broken gauge symmetry had to be investigated. This was done both at the level of gauge independence in R_ξ-gauges, and of unitarity at high energy, known to be delicate for longitudinal gauge bosons. Both constraints led to non-trivial relations among the couplings, which showed that indeed the SM particles exchanges can be cancelled by superpartners exchanges, as heavy as these might be. However, to quantitatively estimate the importance of what remains after this cancellation required one to show that a perturbative expansion of the amplitudes in m_Z/m_χ contained only even powers. Whether such a cancellation can be extended from large s to large $m_\chi = \sqrt{s}/2$ at rest for all diagrams suffering from large s unitarity problems remains an open question.

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Appendix A: Indirect detection energy spectra

The neutrino and photon differential energy spectra of Fig. 1 were extracted from a PYTHIA simulation of 10^6 events for each channel, shown as a function of $x = E_{\nu,\,\gamma}/m_\chi$. For the hard (anti-) neutrinos from the Z boson in the Zh channel that concerned us most, we have been careful to correct the unpolarized PYTHIA results by a factor $\propto x(1-x)$, translating the purely longitudinal polarization of the Z boson, which suppresses forward neutrinos with respect to the WW channel. In spite of this factor, the Zh channel produces the next-to-hardest neutrino spectrum.

For photons, the hard (flat) component of the Zh channel around $x\approx 1$ comes from h loop-decaying into two photons and therefore only appears at the largest values of x. This leaves only a small number of events (~ 10/bin) and large statistical errors which do not appear in the fit shown in the bottom panel of Fig. 1. There are of course no events for $x>1$, but the precise shape of this vanishing (probably similar to that of neutrinos from WW above, as shown) is hidden by these errors.

Appendix B: Dirac and Majorana fermion conventions

To represent the Clifford algebra of the Dirac matrices,

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} ,$$

we use the chiral basis:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} ,$$

where

$$\sigma^\mu = (1, \sigma) , \quad \overline{\sigma}^\mu = (1, -\sigma)$$

are 4D extensions of the Pauli matrices $\sigma = \left(\sigma^1, \sigma^2, \sigma^3\right)$.

For Majorana spinors, we use [25] the usual Dirac Feynman rules after having chosen an arbitrary orientation of the spinor lines. The Majorana condition is

$$\chi = \chi^c = C\overline{\chi}^{\mathrm{T}} , \tag{B.1}$$

and the plane wave expansion of a Majorana field operator is thus [26]

$$\chi(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^{3/2}} \sum_{s=\pm} \left(a_s(p)\, u_s(p)\, \mathrm{e}^{-\mathrm{i}p\cdot x} + a_s^\dagger(p)\, v_s(p)\, \mathrm{e}^{\mathrm{i}p\cdot x}\right) . \tag{B.2}$$

Implementing the Majorana condition (B.1) in (B.2), we find the relations

$$u_s = C\gamma^0 v_s^* , , \tag{B.3}$$
$$v_s = C\gamma^0 u_s^* , \tag{B.4}$$

which allow one to flip the orientation of external Majorana lines.

Appendix C: Neutralino mass matrix

The neutralino mass eigenstates are linear combinations of gaugino and higgsino fields $\left(\tilde{B}, \tilde{W}_3, \tilde{H}_b, \tilde{H}_t\right)$,

$$\chi_i = N_{i1}\tilde{B} + N_{i2}\tilde{W}_3 + N_{i3}\tilde{H}_b + N_{i4}\tilde{H}_t$$

which diagonalize the mass matrix:

$$M = \begin{pmatrix} \begin{matrix} M_1 & 0 \\ 0 & M_2 \end{matrix} & m_Z \times C \\ m_Z \times C^{\mathrm{T}} & \begin{matrix} 0 & -\mu \\ -\mu & 0 \end{matrix} \end{pmatrix} . \tag{C.1}$$

C is the 2×2 electroweak breaking contribution to this neutralino mass matrix:

$$C = \begin{pmatrix} -s_{\mathrm{W}} c_\beta & s_{\mathrm{W}} s_\beta \\ c_{\mathrm{W}} c_\beta & -c_{\mathrm{W}} s_\beta \end{pmatrix} ,$$

with

$$s_{\mathrm{W}} = \sin\theta_{\mathrm{W}}, \quad c_{\mathrm{W}} = \cos\theta_{\mathrm{W}} ,$$
$$s_\beta = \sin\beta, \quad c_\beta = \cos\beta .$$

The normalized eigenvectors can be collected into a unitary matrix N satisfying

$$N^\star M N^{-1} = M_{\mathrm{D}} , \tag{C.2}$$

where M_{D} is a diagonal matrix containing neutralino masses.

In the absence of CP-violating phases, the matrix N can be chosen as a real matrix, and at least one neutralino mass is then negative.

Appendix D: Lagrangian terms for relevant couplings

D.1 Z–χ–χ coupling

We have

$$\mathcal{L} = \frac{1}{2} \sum_{i,j=1}^{4} \overline{\chi_i} \gamma^\mu \left(C_{ij}^V - C_{ij}^A \gamma_5\right) \chi_j Z_\mu ,$$

with

$$C_{ij}^V = \frac{g}{4\cos\theta_{\mathrm{W}}} \left(O_{ij}^Z - O_{ij}^{Z*}\right) , \tag{D.1}$$
$$C_{ij}^A = \frac{g}{4\cos\theta_{\mathrm{W}}} \left(O_{ij}^Z + O_{ij}^{Z*}\right) \tag{D.2}$$

and

$$O_{ij}^Z = N_{i4}N_{j4}^* - N_{i3}N_{j3}^* .$$

In the absence of CP-violation, there is a basis such that $C_{ij}^V = 0$.

D.2 h–Z–Z coupling

We have

$$\mathcal{L} = \frac{1}{2} C_Z h Z_\mu Z^\mu , \tag{D.3}$$

where

$$C_Z = -\frac{g m_Z}{\cos\theta_{\mathrm{W}}} \sin(\alpha - \beta) . \tag{D.4}$$

In practice, we will always work in the decoupling limit $m_A \gg m_Z$, which implies

$$\alpha = \beta - \frac{\pi}{2} . \tag{D.5}$$

Hence, we have $\sin(\alpha - \beta) \simeq -1$ in (D.4).

D.3 h–χ–χ coupling

We have

$$\mathcal{L} = \frac{g}{2} h \overline{\chi}_i \left\{ s_\alpha \left(L^*_{ij} P_L + L_{ij} P_R \right) + c_\alpha \left(K^*_{ij} P_L + K_{ij} P_R \right) \right\} \chi_j \,, \tag{D.6}$$

with

$$K_{ij} = \frac{1}{2} N_{i4} \left(N_{j2} - \tan\theta_{\mathrm{W}} N_{j1} \right) + (i \leftrightarrow j) \,,$$
$$L_{ij} = \frac{1}{2} N_{i3} \left(N_{j2} - \tan\theta_{\mathrm{W}} N_{j1} \right) + (i \leftrightarrow j) \,.$$

The symmetric form of K and L comes from the fact we are working with Majorana particles:

$$\overline{\chi}_i \left(1 \pm \gamma_5\right) \chi_j = \overline{\chi}_j \left(1 \pm \gamma_5\right) \chi_i \,.$$

For real N, this Lagrangian simplifies to

$$\mathcal{L} = \frac{1}{2} \sum_{i,j=1}^{4} C^h_{ij} h \overline{\chi}_i \chi_j \,,$$

where

$$C^h_{ij} = \frac{g}{2\cos\theta_{\mathrm{W}}} O^h_{ij} \tag{D.7}$$

and

$$O^h_{ij} = \left(c_{\mathrm{W}} N_{i2} - s_{\mathrm{W}} N_{i1} \right) \left(s_\alpha N_{j3} + c_\alpha N_{j4} \right) + (i \leftrightarrow j) \,.$$

D.4 Z–fermion–fermion coupling

We have

$$\mathcal{L} = \sum_f \overline{f} \gamma^\mu \left(C^{ZV}_{ff} - C^{ZA}_{ff} \gamma_5 \right) f Z_\mu \,,$$

where

$$C^{ZV}_{ff} = -\frac{g}{2\cos\theta_{\mathrm{W}}} \left(T_{3f} - 2\sin^2\theta_{\mathrm{W}} Q_f \right) \,,$$
$$C^{ZA}_{ff} = -\frac{g}{2\cos\theta_{\mathrm{W}}} T_{3f} \,.$$

Q_f and T_{3f} are the charge and third weak isospin component, with the usual normalization: $T_{3\mathrm{top}} = 1$, and $Q_{\mathrm{top}} = \frac{2}{3}$.

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